



Radar Systems Engineering Lecture 7 Part 2 Radar Cross Section

Dr. Robert M. O'Donnell IEEE New Hampshire Section Guest Lecturer

IEEE New Hampshire Section

IEEE AES Society



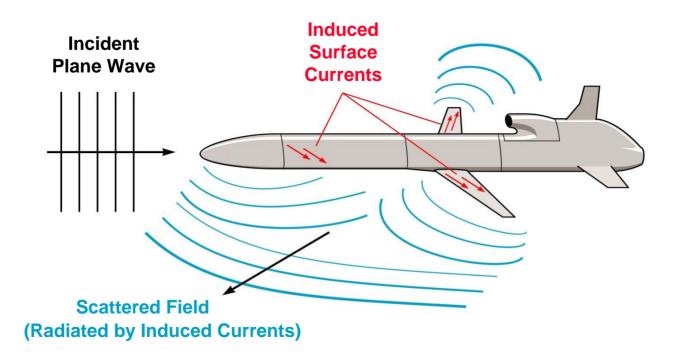
Methods of Radar Cross Section Calculation



RCS Method	Approach to Determine Surface Currents		
Finite Difference-	Solve Differential Form of Maxwell's		
Time Domain (FD-TD)	Equation's for Exact Fields		
Method of Moments	Solve Integral Form of Maxwell's		
(MoM)	Equation's for Exact Currents		
Physical Optics	Currents Approximated by Tangent		
(PO)	Plane Method		
Physical Theory of	Physical Optics with Added Edge		
Diffraction (PTD)	Current Contribution		
Geometrical Optics	Current Contribution Assumed to Vanish		
(GO)	Except at Isolated Specular Points		
Geometrical Theory of	Geometrical Optics with Added Edge		
Diffraction (GTD)	Current Contribution		







- Two step process to determine scattered fields
 - Determine induced surface currents
 - Calculate field radiated by currents

Courtesy of MIT Lincoln Laboratory Used with permission IEEE New Hampshire Section

IEEE AES Society





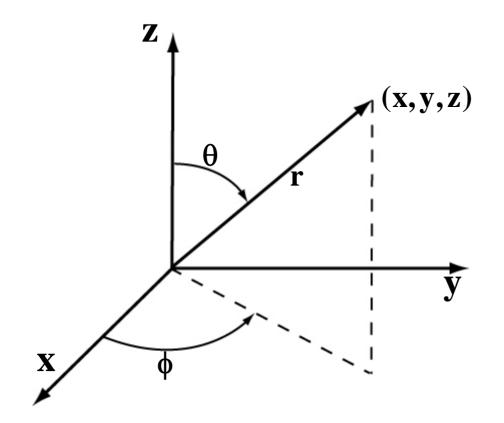
 The Method of Moments calculations predict the exact solution for the target RCS

Surface Patch Model For a Sphere

- Method Solve integral form of Maxwell's Equations
 - Generate a surface patch model for the target -
 - Transform the integral equation form of Maxwell's equations into a set of homogeneous linear equations
 - The solution gives the surface current densities on the target
 - The scattered electric field can then be calculated in a straight forward manner from these current densities
 - Knowledge of the scattered electric field then allows one to readily calculate the radar cross section
- Significant limitations of this method
 - Inversion of the matrix to solve the homogeneous linear equations
 - Matrix size can be very large at high frequencies
 Patch size typically ~λ/10





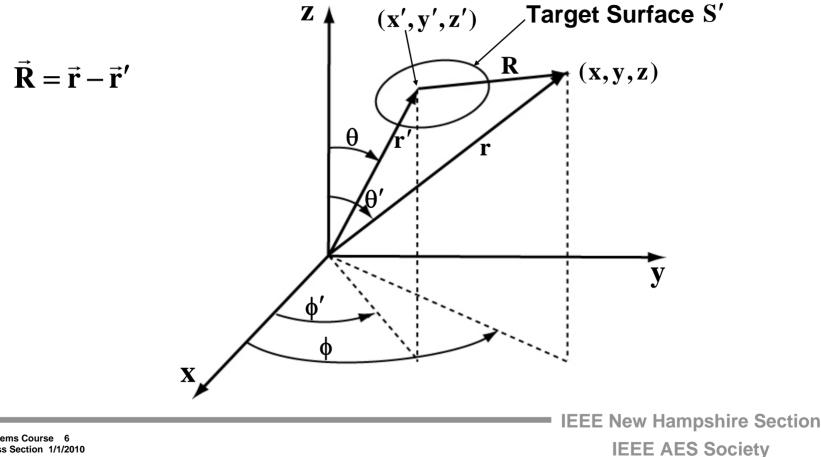




Spherical Coordinate System for MOM Calculations



- Source currents distributed over surface S'
- Field observation point located at (x, y, z)
- Point on surface S' is (x', y', z')







• Maxwell's Equations transform to the Stratton and Chu Equations using the vector Green's Theorem and yield:

$$\vec{\mathbf{E}}_{S} = \oiint_{S'} \left[+ \mathbf{i} \,\omega \,\mu \left(\hat{\mathbf{n}} \, \mathbf{x} \, \vec{\mathbf{H}} \right) \psi + \left(\hat{\mathbf{n}} \, \mathbf{x} \, \vec{\mathbf{E}} \right) \mathbf{x} \, \vec{\nabla} \psi + \left(\hat{\mathbf{n}} \cdot \vec{\mathbf{E}} \right) \vec{\nabla} \psi \, \right] \mathbf{dS'}$$
$$\vec{\mathbf{H}}_{S} = \oiint_{S'} \left[+ \mathbf{i} \,\omega \,\varepsilon \left(\hat{\mathbf{n}} \, \mathbf{x} \, \vec{\mathbf{E}} \right) \psi - \left(\hat{\mathbf{n}} \, \mathbf{x} \, \vec{\mathbf{H}} \right) \mathbf{x} \, \vec{\nabla} \psi - \left(\hat{\mathbf{n}} \cdot \vec{\mathbf{H}} \right) \vec{\nabla} \psi \, \right] \mathbf{dS'}$$
$$\psi = \left[\frac{\mathbf{e}^{+\mathbf{i}\mathbf{k}\mathbf{R}}}{4\pi\,\mathbf{R}} \right] = \begin{array}{c} \text{Free Space} \\ \text{Green's Function} \end{array} \qquad \mathbf{R} = |\mathbf{r} - \mathbf{r'}|$$

• Free space Green's function is an spherical wave falling of as: $_{1/R} \ensuremath{$

• Also, note:
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{I} + \vec{\mathbf{E}}_{S}$$

 $\vec{\mathbf{H}} = \vec{\mathbf{H}}_{I} + \vec{\mathbf{H}}_{S}$





- On the surface of the perfectly conducting target these equations become:
 - Total tangential electric field zero at surface
 - No magnetic sources of currents or charges as source of scattered fields
- Electric Field Integral Equation (EFIE)

$$\vec{\mathbf{E}}_{S} = \oiint_{S'} \left[+\mathbf{i} \ \omega \ \mu (\hat{\mathbf{n}} \ \mathbf{x} \ \vec{\mathbf{H}}) \ \psi + (\hat{\mathbf{n}} \cdot \vec{\mathbf{E}}) \nabla \psi \right] \mathbf{dS'} = \oiint_{S'} \left[+\mathbf{i} \ \omega \ \mu \ \mathbf{J} \ \psi + \frac{1}{\epsilon} \rho \ \nabla \ \psi \right] \mathbf{dS'}$$

• Magnetic Field Integral Equation (MFIE)

$$\vec{\mathbf{H}}_{\mathbf{S}} = \bigoplus_{\mathbf{S}'} (\hat{\mathbf{n}} \mathbf{x} \, \vec{\mathbf{H}}) \mathbf{x} \, \nabla \psi \, \mathbf{dS}' = \bigoplus_{\mathbf{S}'} \vec{\mathbf{J}} \mathbf{x} \, \nabla \psi \, \mathbf{dS}'$$

- Causes of scattered fields
 - Scattered electric field electric currents and charges
 - Scattered magnetic field electric currents





• Applying the boundary conditions for Maxwell's Equations and the Continuity Equation to free space yields:

$$\hat{\mathbf{n}} \mathbf{x} \, \vec{\mathbf{E}}_{\mathrm{I}} = -\,\hat{\mathbf{n}} \mathbf{x} \, \vec{\mathbf{E}}_{\mathrm{S}} = \hat{\mathbf{n}} \mathbf{x} \, \oiint_{\mathrm{S'}} \left[+\,\mathbf{i} \, \omega \, \mu \, \, \vec{\mathbf{J}} \, \psi + \frac{+\,\mathbf{i}}{\omega \, \epsilon} \nabla \cdot \, \vec{\mathbf{J}} \, \nabla \, \psi \right] \mathbf{dS'}$$
$$\hat{\mathbf{n}} \mathbf{x} \, \vec{\mathbf{H}}_{\mathrm{I}} = \frac{\vec{\mathbf{J}}}{2} - \,\hat{\mathbf{n}} \mathbf{x} \, \oiint_{\mathrm{S'}} \vec{\mathbf{J}} \mathbf{x} \, \nabla \, \psi \, \mathbf{dS'}$$

- **Procedure to calculate the scattered electric field:**
 - Convert the integral equation into a set of algebraic equations
 - Solve for induced current density using matrix algebra
 - With the current density known, the calculation of the scattered electric field, \vec{E}^{S} , is reasonably straightforward and the cross section can be calculated: $|\mathbf{r}^{S}|^{2}$

$$\sigma = 4 \pi \mathbf{R}^2 \frac{\left|\mathbf{E}^{\mathbf{S}}\right|^2}{\left|\mathbf{E}^{\mathbf{I}}\right|^2}$$





- Break up the target into a set of N discrete patches
 - 7 to 10 patches per wavelength
- Expand the surface current density as a set of known basis functions

$$\vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sum_{n=1}^{N} \mathbf{I}_n \vec{\mathbf{B}}_n(\vec{\mathbf{r}})$$

Surface Patch Model For Sphere

• Define the "Magnetic Field Operator", $L_{\rm H}(\vec{J})$, as

$$\mathbf{L}_{\mathrm{H}}(\vec{\mathbf{J}}) \equiv \frac{\vec{\mathbf{J}}}{2} - \hat{\mathbf{n}} \mathbf{x} \bigoplus_{\mathbf{S}'} \vec{\mathbf{J}} \mathbf{x} \nabla \psi \, \mathbf{dS}'$$



• Insert the series expansion of currents and bringing the sum out of the operator, we get:

$$L_{\rm H}(\vec{J}) = \sum_{n=1}^{N} I_n L_{\rm H}(\vec{B}_n(\vec{r})) = \hat{n} \times \vec{H}^{\rm I}$$





• Multiply by the weighting vector, $\vec{W}_{\!_m}$, and integrating over the surface:

$$\oint_{S} \left[\vec{W}(\vec{r}) \cdot \left(\hat{n} \ x \ \vec{H}^{I} \right) \right] dS - \sum_{n=1}^{N} I_{n} \ i \ \omega \mu \bigoplus_{S'} \ \bigoplus_{S} \vec{W}_{m} \cdot L(\vec{B}_{n}(r)) dS' dS = 0$$

$$m = 1, 2, 3, \dots N$$

$$- \text{ Point Testing } \vec{W}_{m} = \delta(\vec{r} - \vec{r}_{m})$$

- Galerkin's Method
$$\vec{W}_m = \vec{B}_m(\vec{r})$$

• This is a set of N equations in N unknowns (current coefficients, I_m) of the form:

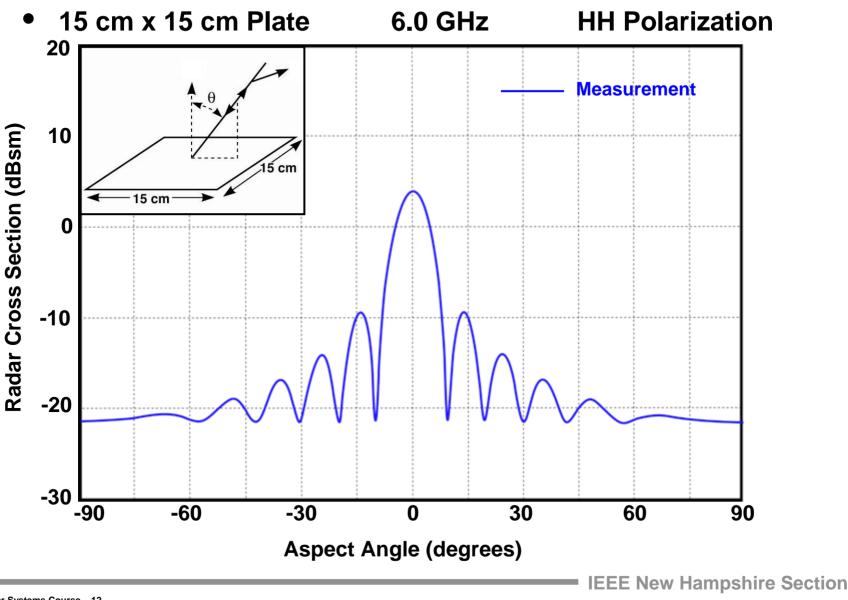
$$\vec{\mathbf{Z}} \, \vec{\mathbf{I}} = \vec{\mathbf{V}} \implies \vec{\mathbf{I}} = \vec{\mathbf{Z}}^{-1} \, \vec{\mathbf{V}}$$

• The only difficulty is inversion of a very large matrix



Monostatic RCS of a Square Plate



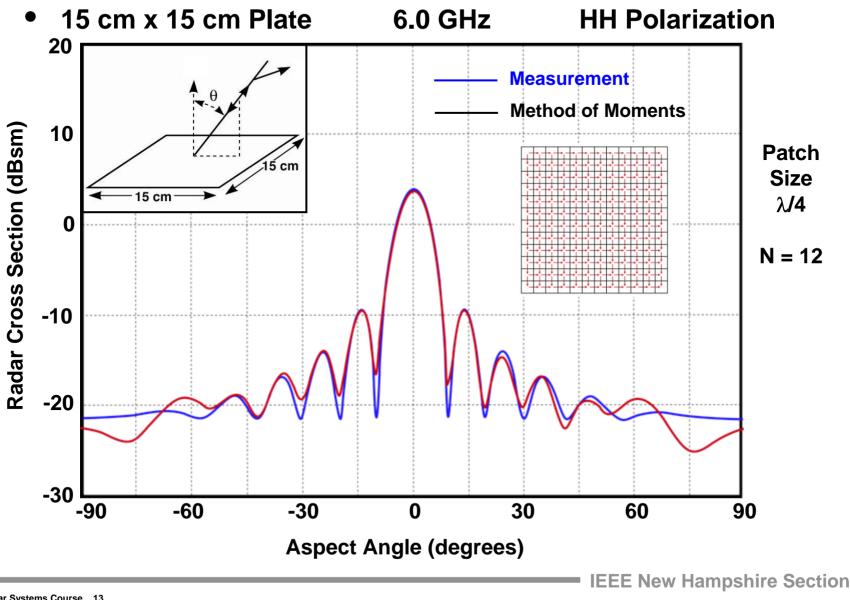


IEEE AES Society



Monostatic RCS of a Square Plate





IEEE AES Society



Surface Patch Model of JGAM for Method of Moments RCS Calculation



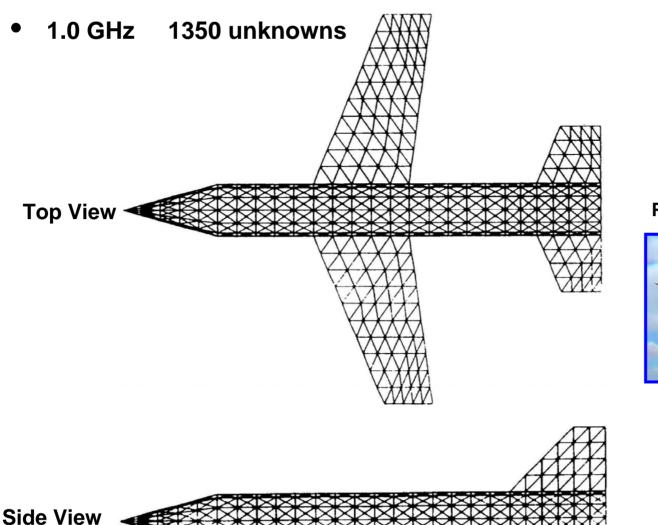
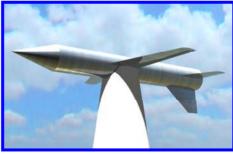


Photo of JGAM on Pylon



Courtesy of MIT Lincoln Laboratory Used with Permission





- Method of moments solution is exact
 - Patch size must be small enough
 - 7 to 10 samples per wavelength
- Well suited for small targets at long wavelengths
 - Example Artillery shell at L-Band (23 cm)
- Aircraft size targets result in extremely large matrices to be inverted
 - JGAM (~ 5m length)
 - 1350 unknowns at 1.0 GHz
 - Typical Fighter aircraft (~ 5m length)
 - A very difficult computation problem at S-Band (10 cm wavelength)

Comparison of MoM and FD-TD Techniques

- For Single Frequency RCS Predictions (perfect conductors)
- 2-Dimensional Calculation
- 3-Dimensional Calculation

	Method of Moments (MoM)	Finite Difference- Time Domain (FD-TD)	
Method of	Integral Equation	Differential Equation	
Calculation	Frequency Domain	Time Domain	
No. of	N (2-D) N ² (3-D)	N^2 (2-D) N ³ (3-D)	
Unknowns			
Memory	Matrix Decomposition	Time Steps	
Requirement	N^{3} (2-D) N^{6} (3-D)	N^3 (2-D) N^4 (3-D)	
Computer	N^2 (2-D) N^4 (3-D)	N^2 (2-D) N^3 (3-D)	
Time			
Accuracy	Exact	Exact	





Methods of Radar Cross Section Calculation



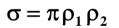
RCS Method	Approach to Determine Surface Currents	
Finite Difference-	Solve Differential Form of Maxwell's	
Time Domain (FD-TD)	Equation's for Exact Fields	
Method of Moments	Solve Integral Form of Maxwell's	
(MoM)	Equation's for Exact Currents	
Geometrical Optics	Current Contribution Assumed to Vanish	
(GO)	Except at Isolated Specular Points	
Physical Optics	Currents Approximated by Tangent	
(PO)	Plane Method	
Geometrical Theory of	Geometrical Optics with Added Edge	
Diffraction (GTD)	Current Contribution	
Physical Theory of	Physical Optics with Added Edge	
Diffraction (PTD)	Current Contribution	





ĥ

- Geometrical Optics (GO) is an approximate method for RCS calculation
 - Valid in the "optical" region (target size >> λ)
- Based upon ray tracing from the radar to "specular points" on the surface of the target
 - "Specular points" are those points, whose normal vector points back to the radar.
- The amount of reflected energy depends on the principal radii of curvature at the surface reflection point
- Geometrical optics (GO) RCS calculations are reasonably accurate to 10 – 15% for radii of curvature of 2 λ to 3λ
- The GO approximation breaks down for flat plates, cylinders and other objects that have infinite radii of curvature; and at edges of these targets



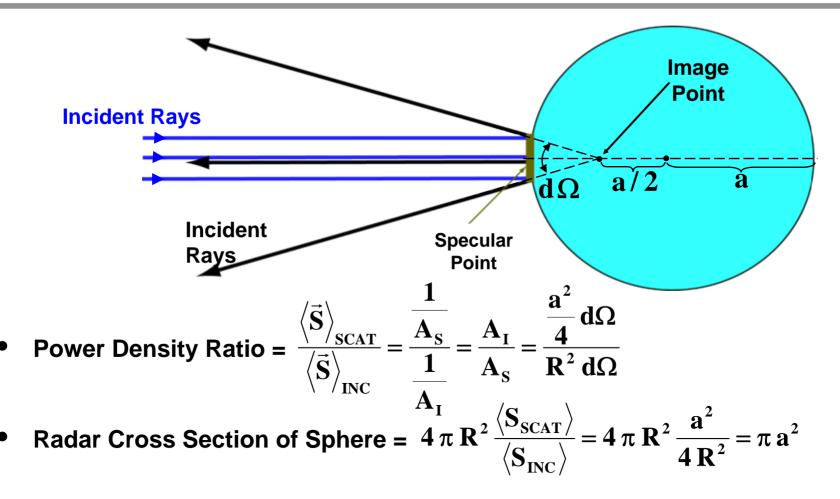
IEEE New Hampshire Section IEEE AES Society

 ρ_1



Geometric Optics

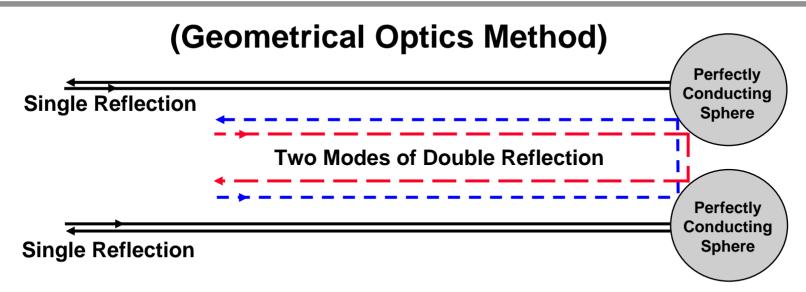




- Radar Cross Section of an Arbitrary Specular Point = $\pi \rho_1 \rho_2$
 - Where radii of curvature at specular point = ρ_1 , ρ_2







- RCS Calculation for Single Reflection
 - Identify all specular points and add contributions
 - Phase calculated from distance to and from specular point
 - Local radii of curvature used to determine amplitude of backscatter
- RCS Calculation for Double Reflection
 - Identify all pairs of specular points
 - At each reflection use single reflection methodology to calculate amplitude and phase



Methods of Radar Cross Section Calculation



RCS Method	Approach to Determine Surface Currents		
Finite Difference-	Solve Differential Form of Maxwell's		
Time Domain (FD-TD)	Equation's for Exact Fields		
Method of Moments	Solve Integral Form of Maxwell's		
(MoM)	Equation's for Exact Currents		
Geometrical Optics	Current Contribution Assumed to Vanish		
(GO)	Except at Isolated Specular Points		
Physical Optics	Currents Approximated by Tangent		
(PO)	Plane Method		
Geometrical Theory of	Geometrical Optics with Added Edge		
Diffraction (GTD)	Current Contribution		
Physical Theory of	Physical Optics with Added Edge		
Diffraction (PTD)	Current Contribution		





- Physical Optics (PO) is an approximate method for RCS calculation
 - Valid in the "optical" region (target size >> λ)
- Method Physical Optics (PO) calculation
 - Modify the Stratton-Chu integral equation form of Maxwell's Equations, assuming that the target is in the far field
 - Assume that the total fields, at any point, on the surface of the target are those that would be there if the target were flat

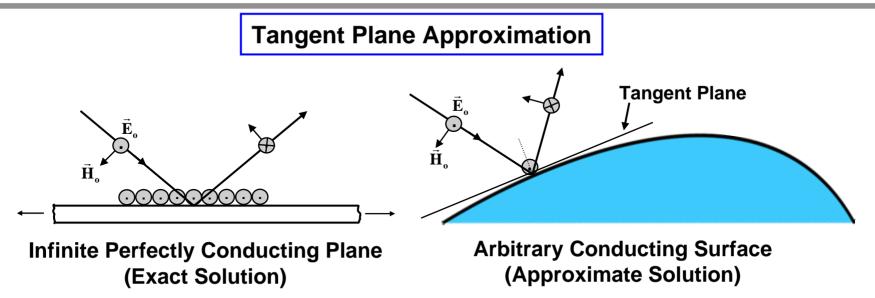
Called "Tangent plane approximation"

- Assume perfectly conducting target
- Resulting equation for the scattered electric field may be readily calculated
- RCS is easily calculated from the scattered electric field
- Physical Optics RCS calculations:
 - Give excellent results for normal (or nearly normal) incidence (< 30°)
 - Poor results for shallow grazing angles and near surface edges
 e.g. leading and trailing edges of wings or edges of flat plates



Physical Optics





• For an incident plane wave :

 $\vec{\mathbf{J}}_{\mathrm{S}}(\vec{\mathbf{r}}') = 2\,\hat{\mathbf{n}}\,\,\mathbf{x}\,\vec{\mathbf{H}}_{\mathrm{o}}\mathbf{e}^{-\mathbf{i}\,\mathbf{k}\,\hat{\mathbf{r}}\,\cdot\,\vec{\mathbf{r}}'}$

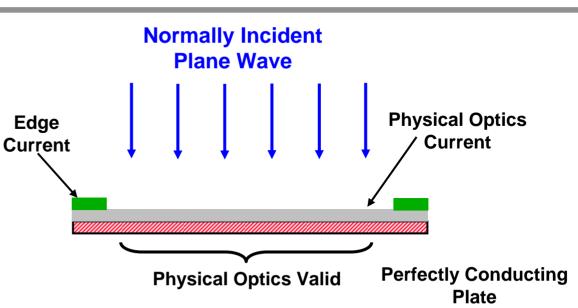
• Substituting this surface current yields (for the monostatic case)

$$\vec{\mathbf{E}}_{\mathrm{S}}(\vec{\mathbf{r}}) = -2i\omega\mu \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{r}}}{4\pi\mathrm{r}} \int \hat{\mathbf{r}} \mathbf{x} \,\hat{\mathbf{r}} \,\mathbf{x} \Big(\hat{\mathbf{n}} \mathbf{x} \,\vec{\mathbf{H}}_{\mathrm{o}}\Big) \mathrm{e}^{-2\,\mathrm{i}\,\mathbf{k}\,\hat{\mathbf{r}}\,\cdot\,\vec{\mathbf{r}}'} \mathrm{d}\vec{\mathbf{r}}'$$

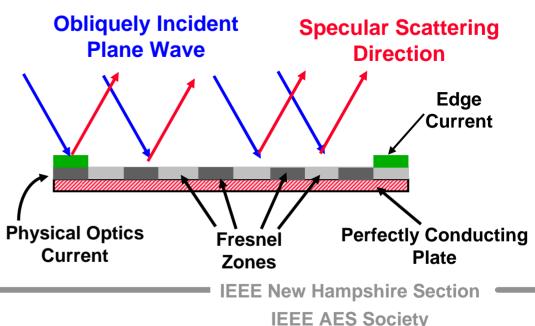




- Physical Optics contribution adds constructively (in phase)
- For large plates, the edge contribution is a small part of the total current
- Except near the edges, Physical Optics gives accurate results

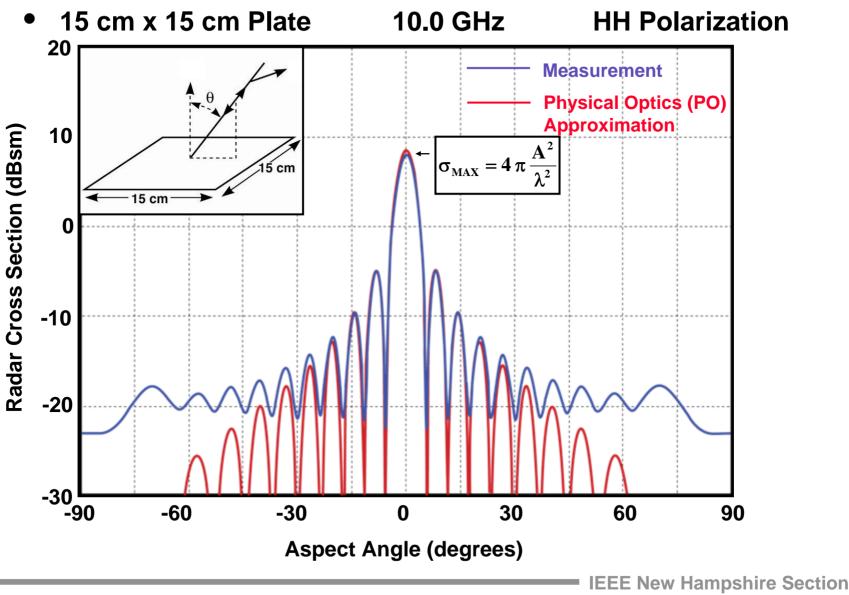


- Except near the edges, Physical Optics gives accurate results
- Fresnel Zones of alternating phase caused by phase delay across plate
- In the backscatter direction, the Physical Optics contribution is predominantly cancelled
- The most significant part of total F current due to edge effects









IEEE AES Society



Methods of Radar Cross Section Calculation



RCS Method	Approach to Determine Surface Currents		
Finite Difference-	Solve Differential Form of Maxwell's		
Time Domain (FD-TD)	Equation's for Exact Fields		
Method of Moments	Solve Integral Form of Maxwell's		
(MoM)	Equation's for Exact Currents		
Geometrical Optics	Current Contribution Assumed to Vanish		
(GO)	Except at Isolated Specular Points		
Physical Optics	Currents Approximated by Tangent		
(PO)	Plane Method		
Geometrical Theory of	Geometrical Optics with Added Edge		
Diffraction (GTD)	Current Contribution		
Physical Theory of	Physical Optics with Added Edge		
Diffraction (PTD)	Current Contribution		



Geometrical Theory of Diffraction (GTD) Overview



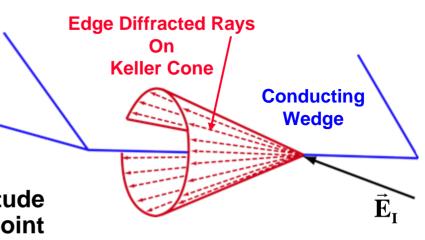
- Geometrical Theory of Diffraction (GTD) a ray tracing method of calculating the diffracted fields at surface edges / discontinuities
 - Assumption: When ray impinges on an edge, a cone (see Keller (1957) Cone below) of diffracted rays are generated
 - Half angle of cone is equal to the angle, β , between the edge and the incident ray.

In backscatter case the cone becomes a disk

- Diffracted electric field proportional to "diffraction coefficients", X and Y and a "divergence factor, Γ , and given by:

$$\left|\vec{\mathbf{E}}_{\mathrm{DIF}}\right| = \frac{\Gamma e^{\mathrm{i}ks} e^{\mathrm{i}\pi/4}}{\sin\beta\sqrt{2\pi ks}} \left(\mathbf{X} \neq \mathbf{Y}\right)$$

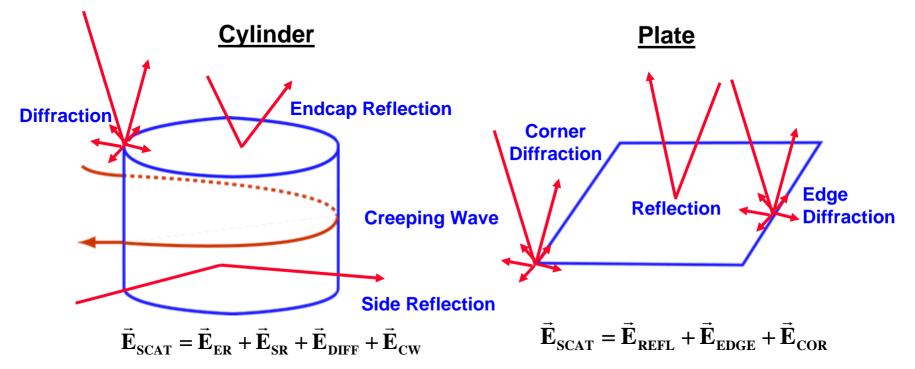
- Diffraction coefficients
 - – when \vec{E}_{I} parallel to edge
 - + when $\vec{\mathbf{H}}_{T}$ parallel to edge
- Divergence factor reduces amplitude as rays diverge from scattering point and accounts for curves edges











- Advantages
 - Easy to Understand
 - Multiple Interactions
- Disadvantages
 - Implementation difficult for complex targets
 - Requires more accurate description than PTD



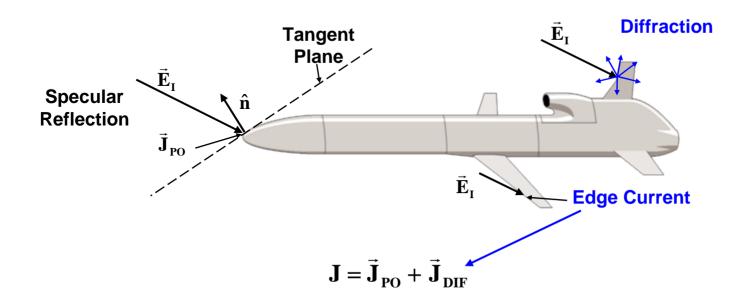
Methods of Radar Cross Section Calculation



RCS Method	Approach to Determine Surface Currents		
Finite Difference-	Solve Differential Form of Maxwell's		
Time Domain (FD-TD)	Equation's for Exact Fields		
Method of Moments	Solve Integral Form of Maxwell's		
(MoM)	Equation's for Exact Currents		
Geometrical Optics	Current Contribution Assumed to Vanish		
(GO)	Except at Isolated Specular Points		
Physical Optics	Currents Approximated by Tangent		
(PO)	Plane Method		
Geometrical Theory of	Geometrical Optics with Added Edge		
Diffraction (GTD)	Current Contribution		
Physical Theory of	Physical Optics with Added Edge		
Diffraction (PTD)	Current Contribution		

Physical Theory of Diffraction (PTD) Overview



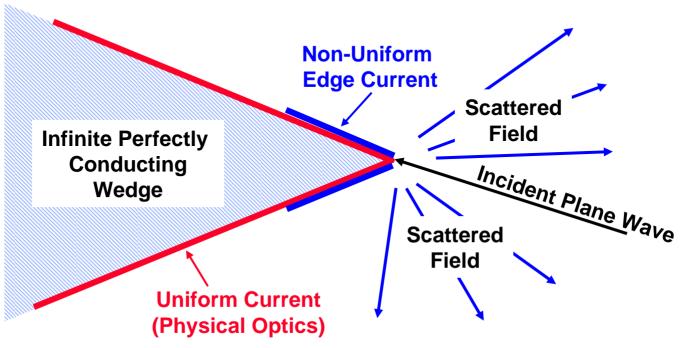


- Approach: Integrate surface current obtained from local tangent plane approximation (plus edge current)
- Advantages: Reduced computational requirements and applicable to arbitrary complex geometries
- Disadvantages: Neglects multiple interactions or shadowing

Courtesy of MIT Lincoln Laboratory Used with permission





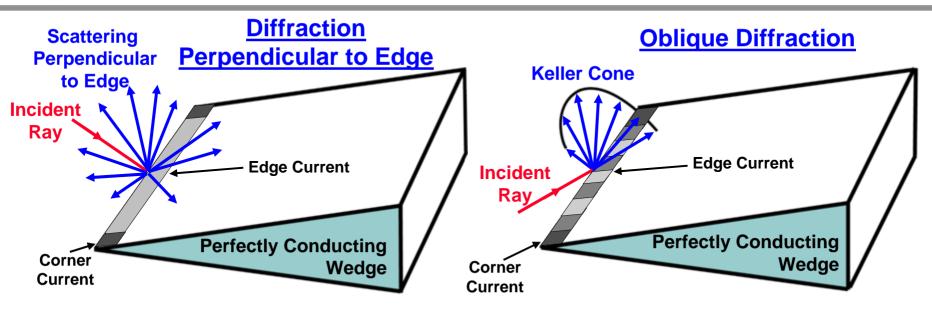


- In 1896, Sommerfeld developed a method to find the total scattered field for an the infinite, perfectly conducting wedge.
- In 1957, Ufimtsev obtained the edge current contributions by subtracting the physical optics contributions from the total scattered field.
- The current for finite length structures may be obtained by truncating the edge current from that of the infinite structure



Normal and Oblique Diffraction





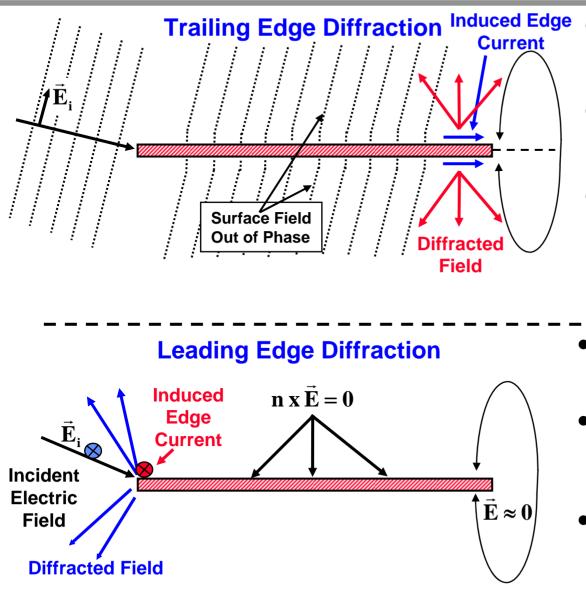
- <u>Constructive</u> addition from edge current contribution along entire edge results in strong perpendicular backscatter
- Small contribution from corner edge current
- Perpendicular to edge, scattering is strong in all directions

- Edge current contribution interferes <u>destructively</u> in direction of backscatter
- For near grazing angles, corner current may be significant
- Strong scattering along "Keller Cone"



Trailing / Leading Edge Diffraction

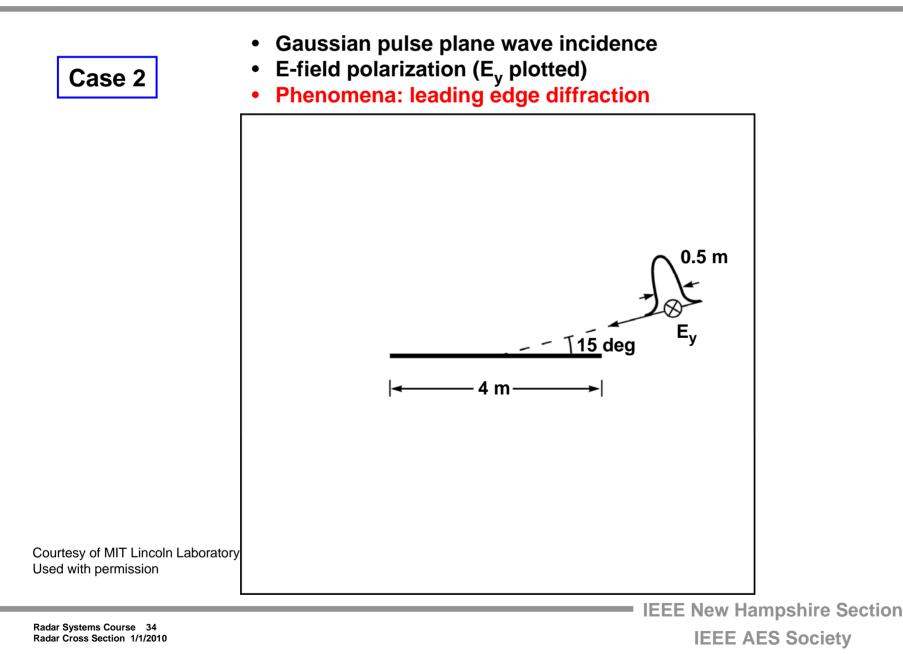




- Negligible scattering at front edge – Electric field normal and continuous
- Traveling waves; above and below plate develop a relative phase delay.
- Required continuity of electric field at back edge causes induced edge current, and thus a diffracted electric field.
- Tangential component of electric field equals zero along the conductor.
- Diffracted electric field is produced by current induced to cancel incident electric field.
- No diffraction at back edge because electric field is close to zero.



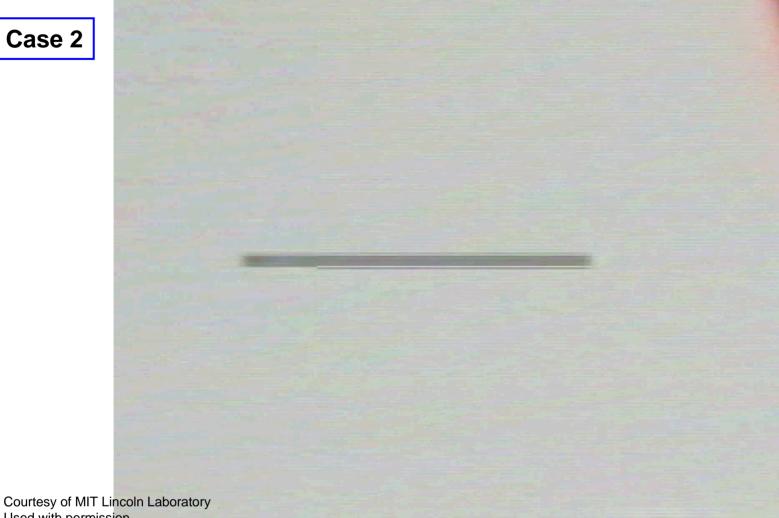








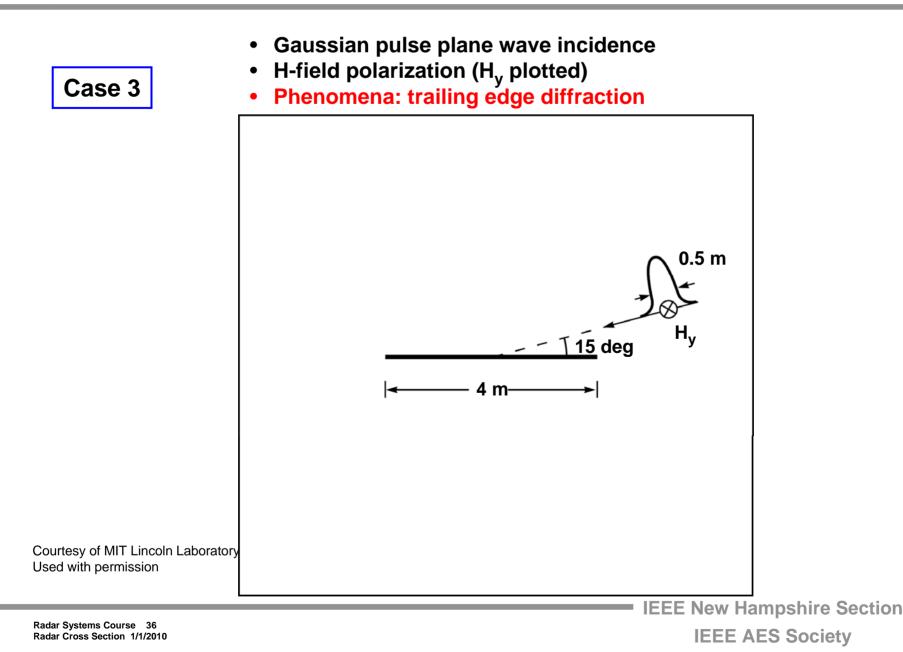
Case 2



Used with permission



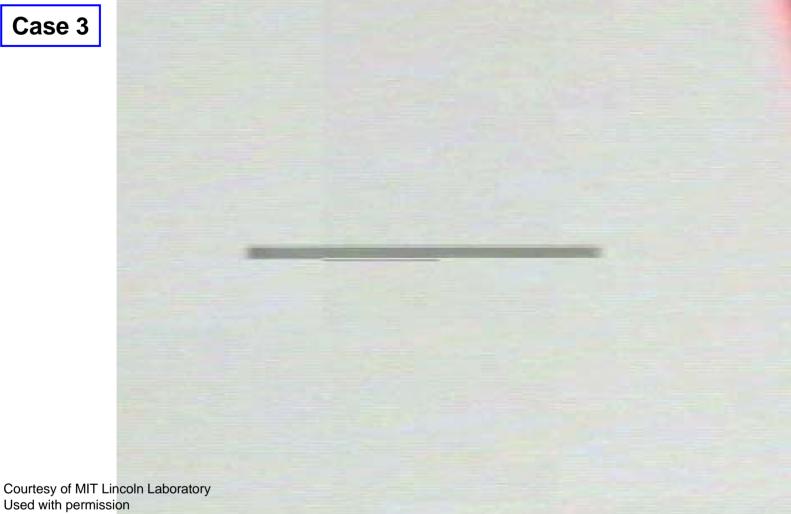






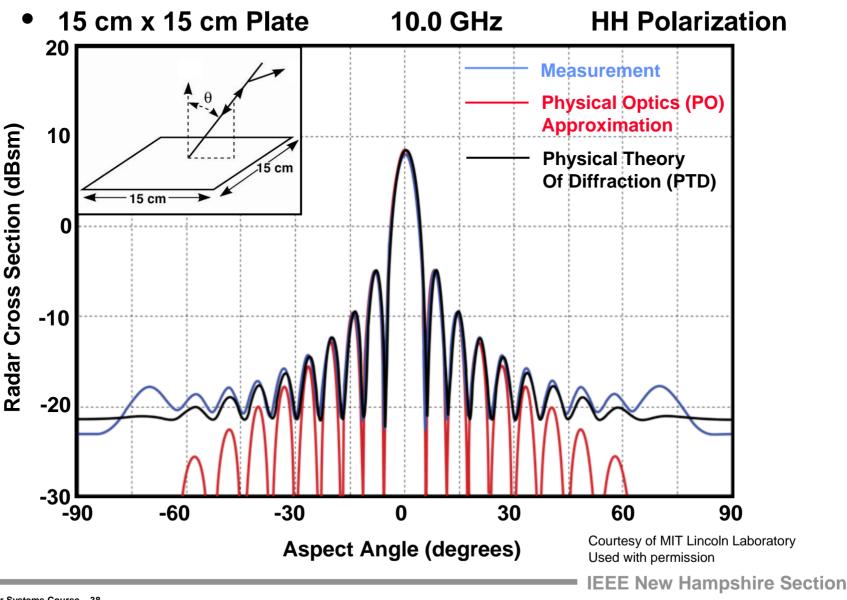












Radar Systems Course 38 Radar Cross Section 1/1/2010

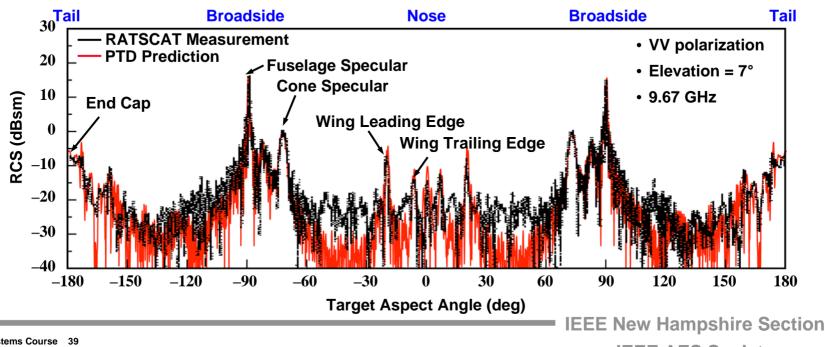
IEEE AES Society







Courtesy of MIT Lincoln Laboratory Used with permission



IEEE AES Society

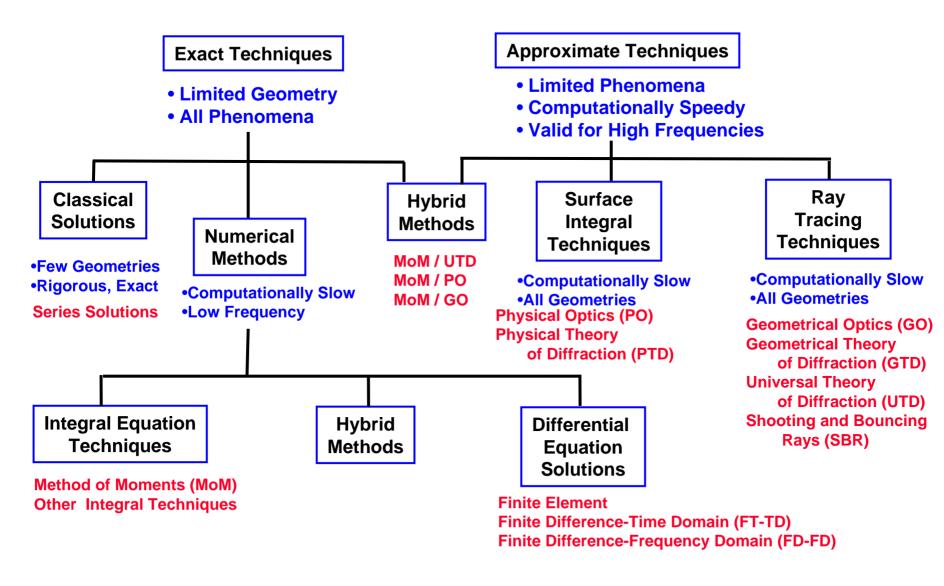




- Introduction
 - A look at the few simple problems
- RCS prediction
 - Exact Techniques
 - Finite Difference- Finite Time Technique (FD-FT) Method of Moments (MOM)
 - Approximate Techniques
 Geometrical Optics (GO)
 Physical Optics (PO)
 Geometrical Theory of Diffraction (GTD)
 Physical Theory of Diffraction (PTD)
- Comparison of different methodologies









Comparison of Different RCS Calculation Techniques



	Methods of Calculation			
	FT-TD	МОМ	GO - GTD	PO-PTD
Calculation Of Current	Exact Solve Partial Differential	Exact (Solve Integral	Specular Point Reflections	Tangent Plane Approximation
Guirein	Equation	Equation)	(Edge Currents)	(Edge Currents)
Physical Phenomena Considered	All	All	Ray Tracing	Reflections (Single & Double) Diffraction
Main Computational Requirement	Time Stepping	Matrix Inversion	Multiple Reflection Diffraction	Surface Integration - Shadowing
Advantages	Exact Visualization Aids Physical Insight	Exact	- Simple Formulation - Good Insight into Physical Phenomena	Easiest Computationally - Good Insight into Physical Phenomena
Limitations And/or	- Low Frequency Only - Complex	- Low Frequency Only - Formulation	- High Frequency Only	- High Frequency Only - Many
Disadvantages	Geometries Difficult - Single Incident Angle	Difficult (Materials)	- Canonical Geometries Only	Phenomena Neglected
		- Single Frequency	- Caustics	



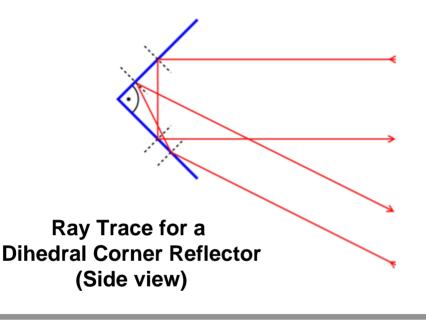


- Give a large reflection, σ , over a wide range of angles
 - Used as test targets and for radar calibration

Square, triangular, and circular

Sailboat Based Circular Trihedral Corner Reflector

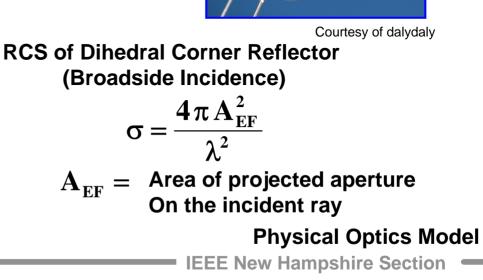




Different shapes

Dihedral

Trihedral



IEEE AES Society





- Target RCS depends on its characteristics and the radar parameters
 - Target : size, shape, material, orientation
 - Radar : frequency, polarization, range, viewing angles, etc
- The target RCS is due to many different scattering centers
 - Structural, Propulsion, and Avionics
- Many RCS calculation tools are available
 - Take into account the many different electromagnetic scattering mechanisms present
- Measurements and predictions are synergistic
 - Measurements anchor predictions
 - Predictions validate measurements





- 1. Atkins, R., *Radar Cross Section Tutorial*, 1999 IEEE National Radar Conference, 22 April 1999.
- 2. Skolnik, M., *Introduction to Radar Systems*, New York, McGraw-Hill, 3rd Edition, 2001.
- 3. Skolnik, M., *Radar Handbook*, New York, NY, McGraw-Hill, 3rd Edition, 2008 (Chapter 14 authored by E. Knott)
- 4. Ruck, et al., *Radar Cross Section Handbook*, Plenum Press, New York, 1970, 2 vols.
- 5. Knott et al., *Radar Cross Section*, Massachusetts, Artech House, Norwood, MA, 1993.
- 6. Bhattacharyya, A. K. and Sengupta, D. L., *Radar Cross* Section Analysis and Control, Artech House, Norwood, MA, 1991.
- 7. Levanon, N., Radar Principles, Wiley, New York, 1988





- Dr. Robert T-I. Shin
- Dr. Robert K. Atkins
- Dr. Hsiu C. Han
- Dr. Audrey J. Dumanian
- Dr. Seth D. Kosowsky





- From Skolnik (Reference 2)
 - Problems 2-10, 2-11, 2-12, and 2-13
- From Levanon (Reference 6)
 - Problems 2-1 and 2-5
- For an ellipsoid of revolution, (semi major axis, a ,aligned with the x-axis, semi minor axis, b, aligned with the y axis, and axis of rotation is the x-axis; what are the radar cross sections (far field) looking down the x, y, and z axes, if the radar has wavelength λ and a >> λ and b >> λ ?
- Extra credit: Solve the last problem assuming a << λ and b << λ .